Permea-TORR

Measures the Air-Permeability of the Cover Concrete and other Porous Materials

Swiss Standard Method SIA 262/1-E

Derivation of Formulae

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Calculation of the Coefficient of Air-Permeability of Concrete kT according to the "Torrent" Method

The derivation that follows was published in [1] and supersedes that originally published in [2].

Assumptions:

1. At the beginning of the test (t=0) all the free pores in the concrete contain air at the atmospheric pressure $\mathbf{P_a}$
2. The concrete is homogeneous and isotropic; in particular its porosity and permeability do not change with the depth from the surface
3. The thickness **e** of the element is equal or larger than the maximum penetration of the test **L**
4. The distribution of pressure across the penetration of the test Y is linear (this is strictly valid only for steady-state conditions)
5. The external chamber "o" (indicated in red in Fig. 1) is always at the same pressure as the inner measurement chamber, and the flow of air into the latter is laminar and perpendicular to the concrete surface

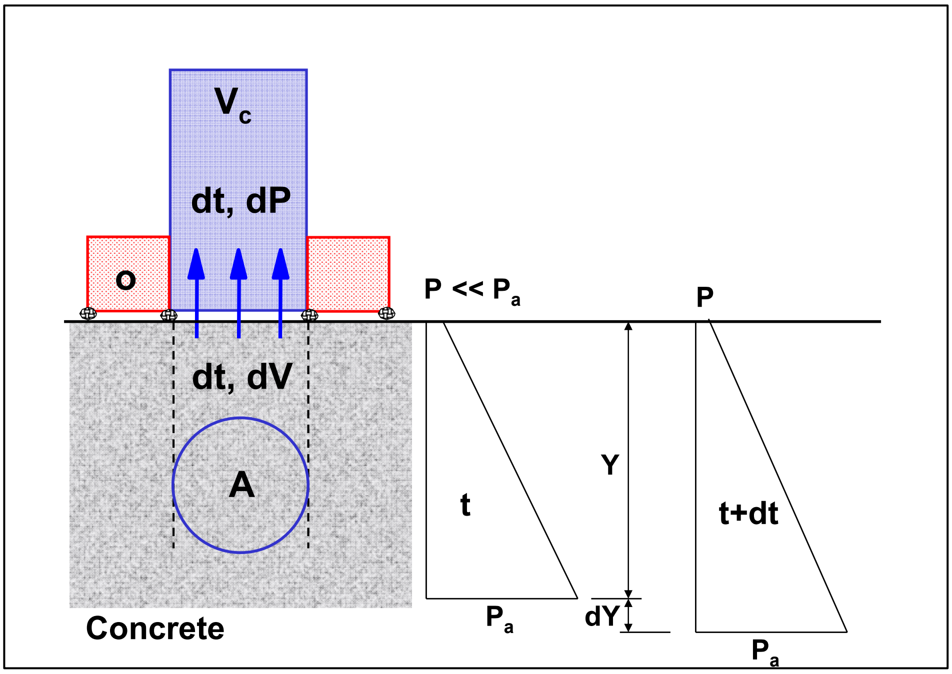

Fig. A.1 – Sketch of the test and principles of the model

After a certain time t of applying a vacuum at the surface, the pores within a certain depth **Y** will have been affected and will contain air at pressures below $\mathbf{P_a}$ (see Fig. A.1).

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[1] Torrent, R. und Frenzer, G., "Methoden zur Messung und Beurteilung der Kennwerte des Ueberdeckungsbetons auf der Baustelle (Teil II)", Office Fédéral des Routes, Rapport No. 516, Bern, Suisse, Oct. 1995, 106 p.

[2] Torrent, R.J., "A two-chamber vacuum cell for measuring the coefficient of permeability to air of the concrete cover on site", Mater. & Struct., v.25, n. 150, July 1992, pp. 358-365.

In the next time interval **dt** a certain volume of air **dV** (**dn** mols) will flow into the inner chamber (of volume V_c), which is at a pressure **P** much smaller than the atmospheric pressure P_a; i.e.:

$$P << P_a \tag{A.1}$$

Also in that interval, the front of pores that are being affected by the vacuum moves deeper by a certain amount **dY**.

The increase in length **dY** of the zone affected by the test will provide the extra mols of air necessary to compensate those lost through the upper surface into the chamber (**dV, dn**).

At time **t** the number of air mols in the pores within the affected zone is

$$n = \frac{P_m \cdot V}{R \cdot T} \tag{A.2}$$

Where:

P_m: mean pressure along length Y [N/m^2]
V: volume of air in the pores within the affected zone [m^3]
R: universal constant of gases [$J . mol^{-1} . K^{-1}$]
T: absolute temperature [K]

Assuming a linear distribution of pressures along **Y**:

$$P_m = (P + P_a) / 2 \tag{A.3}$$

The volume **V** is:

$$V = A \cdot Y \cdot \varepsilon \tag{A. 4}$$

Where:

A: area of the cell's inner chamber
Y: depth of concrete affected by the test at time **t**
ε: porosity of the concrete

replacing (A.3) and (A.4) into (A.2):

$$n = \frac{A \cdot Y \cdot \varepsilon}{R \cdot T} \cdot \frac{Pa + P}{2} \tag{A.5}$$

at time interval **dt** we will have affected an extra amount of mols **dn** due to the increase in depth **dY**

$$dn = \frac{A \cdot \varepsilon}{R \cdot T} \cdot \frac{Pa + P}{2} \cdot dY \tag{A.6}$$

The Hagen-Poiseuille equation for compressible fluids gives the volume of gas **dV** flowing into the chamber under the difference of pressure **(Pa – P)** over a distance **Y**, in a time interval **dt**:

$$dV = \frac{k \cdot A}{2 \cdot \mu} \cdot \frac{P_a^2 - P^2}{P \cdot Y} \cdot dt \qquad \textbf{(A.7)}$$

where:

k: coefficient of permeability [m²]
μ: viscosity of air [N.s/ m²]

transforming into mols and using (A.7):

$$dn = \frac{P \cdot dV}{R \cdot T} = \frac{\cancel{P}}{R \,.\, T} \cdot \frac{k \cdot A}{2 \cdot \mu} \cdot \frac{P_a^2 - P^2}{\cancel{P} \cdot Y} \cdot dt \qquad \textbf{(A.8)}$$

Equating the mols entering the chamber (A.8) with those supplied by the extension of the zone affected by the test (A.6) we have:

$$\frac{A \cdot \varepsilon}{R \cdot T} \cdot \frac{Pa + P}{2} \cdot dY = \frac{k \cdot A}{R \cdot T} \cdot \frac{P_a^2 - P^2}{2 \cdot \mu \cdot Y} \cdot dt \qquad \textbf{(A.9)}$$

rearranging:

$$Y \cdot dY = \frac{k}{\varepsilon \cdot \mu} \cdot (P_a - P) \cdot dt \qquad \textbf{(A.10)}$$

during the test **P** << P_a **, then** $P_a - P \approx P_a$

$$Y \cdot dY = \frac{k}{\varepsilon \,.\, \mu} \cdot P_a \cdot dt \qquad \textbf{(A.11)}$$

integrating:

$$Y^2 / 2 = \frac{k}{\varepsilon \cdot \mu} \cdot P_a \cdot t \qquad \textbf{(A.12)}$$

Then, the penetration **Y** [m] of the test at time **t** [s] is given by:

$$Y = [2 \cdot k \cdot P_a \cdot t / (\varepsilon \cdot \mu)]^{1/2} \qquad \textbf{(A.13)}$$

During the interval **dt**, the volume **dV** of air that enters the inner chamber (of volume V_c) at pressure **P**, increases its pressure by **dP**, where:

$$dP \cdot V_c = P \cdot dV \tag{A.14}$$

Substituting (A.7) into (A.14) and using (A.13):

$$dP \cdot V_c = \frac{k \cdot A}{2 \cdot \mu} \cdot \frac{P_a^2 - P^2}{[2 \cdot k \cdot P_a \cdot t / (\varepsilon \cdot \mu)]^{1/2}} \cdot dt \tag{A.15}$$

$$\frac{dP}{P_a^2 - P^2} = \frac{A}{2 \cdot V_c} \cdot \frac{(k \cdot \varepsilon)^{1/2}}{(2 \cdot \mu \cdot P_a)^{1/2}} \cdot \frac{dt}{t^{1/2}} \tag{A.16}$$

integrating:

$$\frac{1}{2 \cdot P_a} \cdot \ln \frac{P_a + P}{P_a - P} = \frac{A}{V_c} \cdot \frac{(k \cdot \varepsilon)^{1/2}}{(2 \cdot \mu \cdot P_a)^{1/2}} \cdot t^{1/2} \tag{A.17}$$

The limits of integration are:

t_0: initial time = 60 s
P_0: pressure [N/m²] in the inner chamber at initial time **t_0**
t_f: time at the end of the test [s]
P_f: pressure [N/m²] in the inner chamber at final time **t_f**

$$\frac{1}{2 \cdot P_a} \cdot \ln \frac{(P_a + P_f) \cdot (P_a - P_0)}{(P_a - P_f) \cdot (P_a + P_0)} = \frac{A}{V_c} \cdot \frac{(k \cdot \varepsilon)^{1/2}}{(2 \cdot \mu \cdot P_a)^{1/2}} \cdot (t_f^{1/2} - t_0^{1/2}) \tag{A.18}$$

From where we get the formula to calculate k, identified as **kT**:

$$\boxed{kT = \left(\frac{V_c}{A_c}\right)^2 \cdot \frac{\mu}{2 \varepsilon P_a} \left[\frac{\ln\left(\frac{P_a + P_f}{P_a - P_f} \cdot \frac{P_a - P_0}{P_a + P_0} \right)}{\sqrt{t_f} - \sqrt{t_0}} \right]^2} \tag{A.19}$$

kT: coefficient of air-permeability according to “Torrent” method [m²]

Equation A.19 can be further simplified developing the argument of the logarithm:

$$\frac{(P_a + P_f) \cdot (P_a - P_0)}{(P_a - P_f) \cdot (P_a + P_0)} = \frac{P_a^2 + P_a \cdot P_f - P_a \cdot P_0 - \cancel{P_f \cdot P_0}}{P_a^2 - P_a \cdot P_f + P_a \cdot P_0 - \cancel{P_f \cdot P_0}} = \frac{P_a^2 + P_a \cdot (P_f - P_0)}{P_a^2 - P_a \cdot (P_f - P_0)} \tag{A.20}$$

$P_f \cdot P_0$ is negligible compared to the other terms and, by making

$$P_f = P_0 + \Delta P_i \qquad \textbf{(A.21)}$$

where

ΔP_i :   increase in pressure in the inner chamber during the measuring interval t_0-t_f

$$\frac{(P_a + P_f) \cdot (P_a - P_0)}{(P_a - P_f) \cdot (P_a + P_0)} = \frac{P_a + \Delta P_i}{P_a - \Delta P_i} \qquad \textbf{(A.22)}$$

introducing (A.22) in (A.19) we have the formula used by the *Permea-T⊙RR* to compute kT:

$$\boxed{kT = \left(\frac{V_c}{A}\right)^2 \frac{\mu}{2\,\varepsilon\,P_a} \left(\frac{\ln \dfrac{P_a + \Delta P_i}{P_a - \Delta P_i}}{\sqrt{t_f} - \sqrt{t_o}}\right)^2} \qquad \textbf{(A.23)}$$

where:

- kT: coefficient of air-permeability - Torrent Method [m^2]
- V_c : volume of inner cell system [m^3]
- A : cross-sectional area of inner cell [m^2]
- μ : viscosity of air (= 2.0 10^{-5} N.s/m^2)
- ε : estimated porosity of the covercrete (= 0.15)
- P_a : atmospheric pressure [N/m^2]
- ΔP_i: pressure raise in the inner cell at time t_f [N/m^2]
- t_f : time [s] at the end of the test
- t_o : time [s] at the beginning of the test [= 60 s]

The maximum penetration of the test L [m] is computed from (A.13) as:

$$\boxed{L = \left(\frac{2\,kT\,P_a\,t_f}{\varepsilon\,\mu}\right)^{1/2}} \qquad \textbf{(A.24)}$$

Calculation of kT in the Case when the Penetration of the Test Exceeds the Thickness of the Element

The same assumptions, except assumption 3, hold valid for this case. The sketch indicated in Fig. A.1 also applies during the interval $\mathbf{t_0}$ **-** $\mathbf{t_e}$, where $\mathbf{t_e}$ is the time at which the vacuum front reaches the thickness **e** of the element (see Fig. A.2).

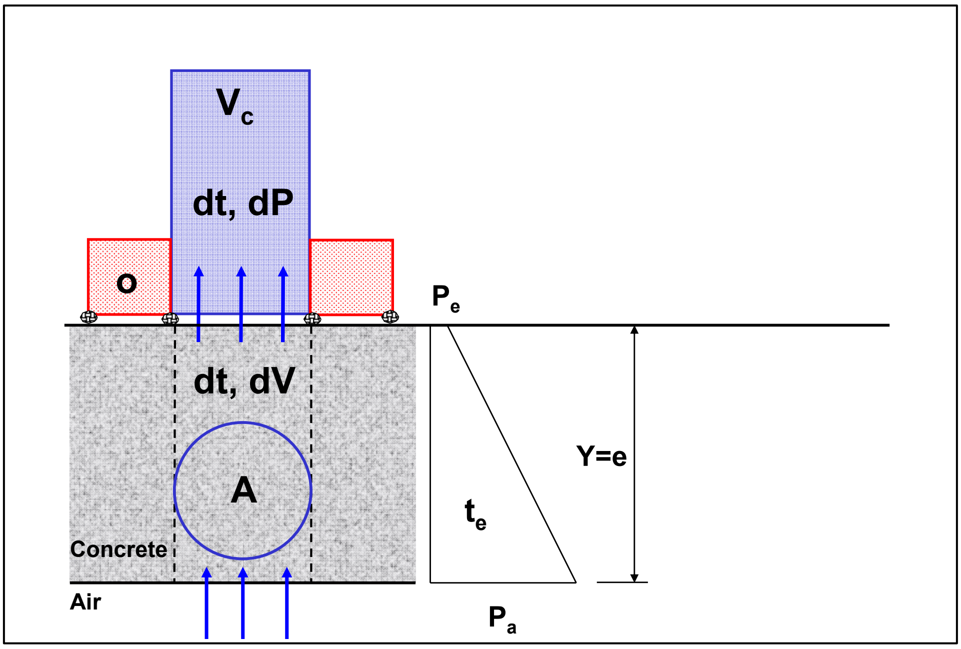

Fig. A.2 – Case when the penetration of the test **Y** exceeds the thickness **e** of the element

In this case, for **t >** $\mathbf{t_e}$, the subsequent increase in pressure in the cell is due to air taken from the other side of the element, across a constant length **e**.

Therefore, two intervals have to be considered:

a) $\mathbf{t_0 - t_e}$, where the flow takes place across a growing length **Y** (equations derived above hold valid)
b) $\mathbf{t_e - t_f}$, where the flow takes place across a constant length **Y = e**

Now, equation A.15 will contain two terms, the first valid for interval a) and the second for interval b):

For interval a)

$$dP \cdot V_c = \frac{k \cdot A}{2 \cdot \mu} \cdot \frac{P_a^2 - P^2}{[2 \cdot k \cdot P_a \cdot t / (\varepsilon \cdot \mu)]^{1/2}} \cdot dt + \frac{k \cdot A}{2 \cdot \mu} \cdot \frac{P_a^2 - P^2}{e} \cdot dt \qquad \text{(A.25)}$$

or

$$\frac{dP}{P_a^2 - P^2} = \frac{A}{2 \cdot V_c} \cdot \frac{(k \cdot \varepsilon)^{½}}{(2 \cdot \mu \cdot P_a)^{½}} \cdot \frac{dt}{t^{½}} + \frac{k \cdot A}{2 \cdot V_c \cdot \mu \cdot e} \cdot dt \qquad \text{(A.26)}$$

or

$$\frac{V_c}{A} \cdot \frac{dP}{P_a^2 - P^2} = \frac{(k \cdot \varepsilon)^{½}}{2 \cdot (2 \cdot \mu \cdot P_a)^{½}} \cdot \frac{dt}{t^{½}} + \frac{k}{2 \cdot \mu \cdot e} \cdot dt \qquad \text{(A.27)}$$

integrating (A.27):

$$\frac{V_c}{2 \cdot A \cdot P_a} \cdot \ln \frac{P_a + P}{P_a - P} = \frac{(k \cdot \varepsilon)^{½}}{(2 \cdot \mu \cdot P_a)^{½}} \cdot t^{½} + \frac{k}{2 \cdot \mu \cdot e} \cdot dt \qquad \text{(A.28)}$$

The limits of integration for the first member are:

t_0: initial time = 60 s
P_0: pressure [N/m²] in the inner chamber at initial time **t_0**
t_f: time at the end of the test [s]
P_f: pressure [N/m²] in the inner chamber at final time **t_f**

which are the same as for the general case, giving the same result as equation (A.18)

$$\frac{V_c}{2 \cdot A \cdot P_a} \cdot \ln \frac{(P_a + P_f) \cdot (P_a - P_0)}{(P_a - P_f) \cdot (P_a + P_0)} = \frac{(kT \cdot \varepsilon)^{½}}{(2 \cdot \mu \cdot P_a)^{½}} \cdot (t_f^{½} - t_0^{½}) \qquad \text{(A.29)}$$

Notice that the value **kT** that appears in the second member is the value given by the instrument, under the assumption that **L < e**.

Integrating the second member of equation (A.28) we have two terms; the first term, corresponding to interval a), is to be integrated within the following limits:

t_0: initial time = 60 s
t_e: time [s] when the vacuum front reaches the thickness of the element [s]

The second term, corresponding to interval b), is to be integrated within the following limits:

t_e: time [s] when the vacuum front reaches the thickness of the element [s]
P_e: pressure [N/m²] in the inner chamber at time **t_e**
t_f: time at the end of the test [s]
P_f: pressure [N/m²] in the inner chamber at final time **t_f**

The second member of (A.27) results, after integration of both terms:

$$\frac{(kT_e \cdot \varepsilon)^{½}}{(2 \cdot \mu \cdot P_a)^{½}} \cdot (t_e^{½} - t_0^{½}) + \frac{kT_e}{2 \cdot \mu \cdot e} \cdot (t_f - t_e) \qquad \text{(A.30)}$$

where:

kT_e: coefficient of air permeability [m^2], corrected for thickness of the element

now, from (A.13) we can calculate the value of **t_e** as:

$$t_e = \frac{e^2 \cdot \varepsilon \cdot \mu}{kT_e \cdot 2 \cdot P_a} \qquad \textbf{(A.31)}$$

substituting (A.31) in (A.30) and equating to the second member of (A.29)

$$\frac{(kT_e \cdot \varepsilon)^{½}}{(2 \cdot \mu \cdot P_a)^{½}} \cdot \left[\frac{e \cdot (\varepsilon \cdot \mu)^{½}}{(kT_e \cdot 2 \cdot P_a)^{½}} - t_0^{½} \right] + \frac{kT_e}{2 \cdot \mu \cdot e} \cdot \left(t_f - \frac{e^2 \cdot \varepsilon \cdot \mu}{kT_e \cdot 2 \cdot P_a}\right) = \frac{(kT \cdot \varepsilon)^{½}}{(2 \cdot \mu \cdot P_a)^{½}} \cdot (t_f^{½} - t_0^{½}) \qquad \textbf{(A.32)}$$

reorganizing:

$$\frac{P_a \cdot t_f}{\mu \cdot e} \cdot kT_e - \frac{(2 \cdot P_a \cdot \varepsilon \cdot t_0)^{½}}{\mu^{½}} \cdot kT_e^{½} + \frac{e \cdot \varepsilon}{2} \cdot \frac{(2 \cdot P_a \cdot \varepsilon \cdot kT)^{½}}{\mu^{½}} \cdot (t_f^{½} - t_0^{½}) = 0 \qquad \textbf{(A.33)}$$

or:

$$A \cdot kT_e + B \cdot kT_e^{½} + C = 0$$

where:

$$A = \frac{P_a \cdot t_f}{\mu \cdot e}$$

$$B = -\frac{(2 \cdot P_a \cdot \varepsilon \cdot t_0)^{½}}{\mu^{½}}$$

$$C = \frac{e \cdot \varepsilon}{2} \cdot \frac{(2 \cdot P_a \cdot \varepsilon \cdot kT)^{½}}{\mu^{½}} \cdot (t_f^{½} - t_0^{½})$$

which allows us, by solving the 2nd degree equation, and squaring the result, to get the coefficient of air-permeability **kT_e**, corrected for thickness:

$$kT_e = \frac{[-B + (B^2 - 4 \cdot A \cdot C)^{½}]^2}{4 \cdot A^2}$$

The calculation is done automatically by *PermeaTORR*'s software by pressing the key L>e? on the touch-screen panel and entering the value of **e** in mm. For those using Proceq's "Torrent Permeability Tester", an Excel file can be ordered free of charge from info@m-a-s.com.ar.